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Grand Test	HSC EXAMINATION SET - A HINT & SOLUTION MATHEMATICS	M.Marks : 80 Duration : 3 Hrs

Q.1A (i) (p V q) → (q v r) ≡ (T v F) → (F v T) ≡ T → T ≡ T

$\therefore$ truth value of the given statement is T. Ans (a)

(ii) Comparing the given equation with $ax^2+2hxy+by^2+2gx+2fy+c=0$,

we get, a = 3, h = 5, b = 3, g = 0, f = 8, c = k.

Now, given equation represents a pair of lines. $\therefore abc+2fgh-af^2-bg^2-ch^2=0$

$\therefore$ (3)(3)(k) + 2 (8)(0)(5) – 3 (8)2 − 3(0)2 – k(5)2 = 0

$\therefore$ 9k + 0 – 192 − 0 − 25k = 0

$\therefore$ −16k – 192 = 0, $\therefore$ −16k = 192, $\therefore$ k = −12. Ans. (b)

(iii) we know that

$$\tan\frac{\pi}{3}=\sqrt{3} \quad and \quad \tan(\pi+x)=\tan x$$

$$\tan\frac{\pi}{3}=\tan\left(\pi+\frac{\pi}{3}\right)=\tan\frac{4\pi}{3}$$

$$\tan\frac{\pi}{3}=\tan\frac{4\pi}{3}=\sqrt{3} \quad where \quad 0<\frac{\pi}{3}<2\pi \quad and \quad 0<\frac{4\pi}{3}<2\pi$$

$$\tan x=\sqrt{3} \quad gives \quad \tan x=\tan\frac{\pi}{3}=\tan\frac{4\pi}{3}$$

$$x=\tan\frac{\pi}{3} \quad and \quad x=\frac{4\pi}{3} \quad \text{Ans(b)}$$

Q.1.(B) (i) If $\bar{r}\ .\ \bar{n}=d_1$ and $\bar{r}\ .\ \bar{n}=d_2$ are two parallel planes then the distance between them is equal to

$$\left|\frac{d_1-d_2}{\sqrt{a^2+b^2+c^2}}\right| \quad \text{Here } d_1=5 \quad and \quad d_2=\frac{-20}{3}$$

$$=\left|\frac{5-\left(\frac{-20}{3}\right)}{\sqrt{4+9+36}}\right|=\left|\frac{35}{3\sqrt{49}}\right|=\frac{5}{3}$$

(ii) Comparing the equation $4x^2 + 5xy + y^2 = 0$ with $ax^2 + 2hxy + by^2 = 0$,

We get a = 4, 2h = 5, h = (5/2), b = 1, Let θ be the acute angle between the lines

$$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \left|\frac{2\sqrt{\left(\frac{5}{2}\right)^2 - 4(1)}}{4+1}\right| = \left|\frac{2\sqrt{\left(\frac{25}{4}\right) - 4}}{4+1}\right| = \left|\frac{2x\frac{3}{2}}{5}\right| = \left|\frac{3}{5}\right| \qquad \therefore \theta = \tan^{-1}\left(\frac{3}{5}\right)$$

(iii) Negation of $r \to (\sim p \wedge q) \quad = r \wedge \ \sim(\sim p \wedge q) \quad = r \wedge (p \vee \sim q)$

(iv) The position vectors $\bar{a}$ *and* $\bar{b}$ of the points A and B are $\bar{a} = 2\bar{i} - \bar{j} + 5\bar{k}$ *and* $\bar{b} = -3\bar{i} + 2\bar{j}$

Let C be the point, with position vectors $\bar{c}$ divides the line segment AB internally in the ratio 1 : 4.

$\therefore$ by the section formula for internal division,

$$\bar{c} = \frac{1\ .\ \bar{b} + 4\ .\ \bar{a}}{1+4} = \frac{\left(-3\bar{i}+2\bar{j}\right)+4\left(2\bar{i}-\bar{j}+5\bar{k}\right)}{5} = \frac{1}{5}\left(5\bar{i}-2\bar{j}+20\bar{k}\right), \qquad \bar{c} = \bar{i} - \frac{2}{5}\bar{j} + 4\bar{k}$$

(v) The Cartesian equation of a line is $\frac{x+5}{3} = \frac{y+4}{5} = \frac{z+5}{6}$

$\therefore$ The line is passing through the point A (-5, - 4, - 5) and having direction ratios 3, 5, 6.

Let $\bar{a}$ be the position vector of A w.r.t. the origin and $\bar{b}$ be the vector parallel to the line.

$\bar{a} = -5\bar{i} - 4\bar{j} - 5\bar{k}$ *and* $\bar{b} = 3\bar{i} + 5\bar{j} + 6\bar{k}$

The vector form of the equation of line passing through A ($\bar{a}$) and parallel to $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$ where, λ is a scalar.

$\therefore$ The vector form of the equation of given line is $\bar{r} = \left(-5\bar{i} - 4\bar{j} - 5\bar{k}\right) + \lambda\left(3\bar{i} + 5\bar{j} + 6\bar{k}\right)$

Q.2 (A) (i) $\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$

This is of the form AX = B, where $A = \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ & $B = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$

To Find A^{-1} , $|A| = \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix} = 2 - 9 = -7 \neq 0$, $\therefore$ A^{-1} exists

To Find A^{-1} , we find cofactor matrix

$A_{11} = (-1)^{1+1}(1) = 1$,, $A_{12} = (-1)^{1+1}(3) = -3$, $A_{21} = (-1)^{2+1}(3) = -3$, $A_{22} = (-1)^{2+2}(2) = 2$

$\therefore \textit{Cofactor Matrix} = \begin{bmatrix} 1 & -3 \\ -3 & 2 \end{bmatrix}$, $\therefore \textit{Adj. } A = \begin{bmatrix} 1 & -3 \\ -3 & 2 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{-7}\begin{bmatrix} 1 & -3 \\ -3 & 2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -1 & 3 \\ 3 & -2 \end{bmatrix}$,

Pre-multiply AX = B by A $^{-1}$, $\therefore (A^{-1}A)X = A^{-1}B$, $\therefore IX = A^{-1}B$,

$$\therefore x = \begin{bmatrix} \frac{-1}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{-2}{7} \end{bmatrix} \begin{bmatrix} -5 \\ 3 \end{bmatrix}, \quad \therefore x = \begin{bmatrix} \frac{5}{7} & +\frac{9}{7} \\ \frac{-15}{7} & \frac{-6}{7} \end{bmatrix} \begin{bmatrix} \frac{14}{7} \\ \frac{-21}{7} \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix},$$

By equality of matrices x = 2, y = 3 is the required solutions.

ii) We find the joint equation of the pair of lines OA and OB through origin, each making an angle of 60^0 with x + y = 10 whose slope is −1.

Let OA (or OB) has slope m. ,∴ its equation is y = mx ....(1)

Also, $\tan 60 = \left|\frac{m-(-1)}{1+m(-1)}\right|$, $\therefore \sqrt{3} = \left|\frac{m+1}{1-m}\right|$, Squaring both sides, we get,

$$\therefore 3 = \frac{(m+1)^2}{(m-1)^2} \qquad \therefore 3(1-2m+m^2) = m^2+2m+1$$

$$\therefore 3-6m+3m^2 = m^2+2m+1, \qquad \therefore 2m^2-8m+2=0$$

$$\therefore m^2-4m+1=0$$

$$\therefore \left(\frac{y}{x}\right)^2 - 4\left(\frac{y}{x}\right)+1 = 0 \text{..............(1)}$$

$$\therefore y^2-4xy+x^2=0$$

$\therefore x^2-4xy+y^2=0$ is the joint equation of the two lines through the origin each making an angle of 60^0 with x + y = 10

$\therefore x^2-4xy+y^2=0$ *and* $x+y=10$ form a triangle OAB which is equilateral.

(iii) The lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ & $\frac{x-x_2}{l_1} = \frac{y-y_2}{m_1} = \frac{z-z_2}{n_1}$ intersect if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_1 \end{vmatrix} = 0$

The equation of the given lines are $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ & $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$

$\therefore x_1=1, y_1=-1, z_1=1 \,\&\, x_2=3, y_2=k, z_2=0$, $\therefore l_1=2, m_1=3, n_1=4 \,\&\, l_2=1, m_2=2, n_2=1$

Since these lines intersect, we get $\begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$\therefore 2(3-8)-(k+1)(2-4)-1(4-3)=0$, $\therefore -10+2(k+1)-1=0$, $\therefore (k+1) = \frac{11}{2}$, $\therefore (k) = \frac{9}{2}$

Q.2(B) (i):

p	Q	r	~ p	~ q	$p \vee \sim q$	$\sim p \wedge q$	$(p\vee \sim q) \vee (\sim p \wedge q)$	$(I) \wedge r$
T	T	T	F	F	T	F	T	T
T	T	F	F	F	T	F	T	F
T	F	T	F	T	T	F	T	T
T	F	F	F	T	T	F	T	F
F	T	T	T	F	F	T	T	T
F	T	F	T	F	F	T	T	F
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	F

The entries in the last column are neither all T nor all F. $[(\therefore$ p $\vee \sim$ q$) \vee (\sim$ p $\wedge$ q$)] \wedge$ r is a contingency.

(ii) cos A = sin B – cos C, cos A + cos C = sin B

$$\therefore 2\cos\left(\frac{A+C}{2}\right).\cos\left(\frac{A-C}{2}\right) = \sin B \quad , \quad \therefore 2\cos\left(\frac{\pi}{2}-\frac{B}{2}\right).\cos\left(\frac{A-C}{2}\right) = \sin B \quad [\because A+B+C=\pi]$$

$$\therefore 2\sin\frac{B}{2}.\cos\left(\frac{A-C}{2}\right) = 2\sin\frac{B}{2}.\cos\frac{B}{2}$$

$$\therefore \cos\left(\frac{A-C}{2}\right) = \cos\frac{B}{2} \;, \quad \therefore \left(\frac{A-C}{2}\right) = \frac{B}{2} \;, \quad \therefore A-C=B \;, \therefore A = B + C$$

$\therefore$ A +B + C = 180 ^{0} gives, A + A = 180 ^{0} , $\therefore$2A = 180 ^{0} $\therefore$ A = 90 ^{0} , , $\therefore\triangle$ABC is a right-angled triangle

(iii) The line of the vector $\bar{u}$ is equally inclined to the coordinate axes.

If α, β, γ are the direction angles of this, then $\alpha = \beta = \gamma$

$$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \text{ gives } \therefore 3\cos^2\alpha = 1, \therefore \cos^2\alpha = 1/3 \quad \therefore \cos\alpha = \pm\left(1/\sqrt{3}\right)$$

$\therefore$ The unit vectors along this line are

$$\left(1/\sqrt{3}\right)\bar{i} + \left(1/\sqrt{3}\right)\bar{j} + \left(1/\sqrt{3}\right)\bar{k} \quad and \quad \left(-1/\sqrt{3}\right)\bar{i} + \left(-1/\sqrt{3}\right)\bar{j} + \left(-1/\sqrt{3}\right)\bar{k}$$

i.e., $\left(1/\sqrt{3}\right)\left(\bar{i}+\bar{j}+\bar{k}\right)$ and $\left(-1/\sqrt{3}\right)\left(\bar{i}+\bar{j}+\bar{k}\right)$

Now $|\bar{u}| = 3$, $|\bar{u}| = \pm 3\left(1/\sqrt{3}\right)\left(\bar{i}+\bar{j}+\bar{k}\right)$, $i.e. |\bar{u}| = \pm\sqrt{3}\left(\bar{i}+\bar{j}+\bar{k}\right)$

Q.3(A) (i) Let the cost of 1 dozen of pencils, 1 dozen of pens and 1 dozen of erasers be Rs. x, Rs. y, and Rs. z respectively. Then, from the given conditions,

4x + 3y + 2z = 60

2x + 4y + 6z = 90, i.e., x + 2y + 3z = 45

6x + 2y + 3z = 70

These equations can be written in the matrix form as :

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

$$\begin{bmatrix}1&2&3\\4&3&2\\6&2&3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}60\\45\\70\end{bmatrix} \text{ By } R_1 \leftrightarrow R_2$$

$$\begin{bmatrix}1&2&3\\0&-5&-10\\0&-10&-15\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}45\\-120\\-200\end{bmatrix}, \text{ By } R_2-4R_1 \quad \textit{and} \quad R_2-6R_1$$

$$\begin{bmatrix}1&2&3\\0&-5&-10\\0&0&5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}45\\-120\\40\end{bmatrix} \text{ By } R_3-2R_2$$

$$\begin{bmatrix}x+2y+3z\\0-5y-10z\\0+0+5z\end{bmatrix}=\begin{bmatrix}45\\-120\\40\end{bmatrix}$$

By equality of matrices,
x + 2y + 3z = 45 …(1), − 5y –10z = − 120 …(2),   5z = 40 …(3)
From (3), z = 8
Substituting z = 8 in (2), we get -5y – 80 = -120 $\therefore$ - 5y = - 40 $\therefore$ y = 8
Substituting y = 8, z = 8 in (1), we get, x + 16 + 24 = 45 $\therefore$ x + 40 = 45 $\therefore$ x = 5
$\therefore$ x = 5, y = 8, z = 8

Hence, the cost is Rs. 5 per dozen for pencils, Rs. 8 per dozen for pens and Rs. 8 per dozen of erasers.

(ii) $\bar{a}=2\bar{i}+\bar{j}-4\bar{k}$ *and* $\bar{b}=2\bar{i}-\bar{j}+3\bar{k}$, $\bar{c}=3\bar{i}+\bar{j}-2\bar{k}$ *and* $\bar{p}=-\bar{i}-3\bar{j}+4\bar{k}$

Suppose $\bar{p}=x\bar{a}+y\bar{b}+z\bar{c}$

Then $-\bar{i}-3\bar{j}+4\bar{k}=x(2\bar{i}+\bar{j}-4\bar{k})+y(2\bar{i}-\bar{j}+3\bar{k})+z(3\bar{i}+\bar{j}-2\bar{k})$

$\therefore -\bar{i}-3\bar{j}+4\bar{k}=\bar{i}(2x+2y+3z)+\bar{j}(x-y+z)+\bar{k}(-4x+3y-2z)$

By equality of vectors, 2x + 2y + 3z = - 1, x – y + z = - 3, -4 x + 3y – 2z = 4
We have to solve these equations by using Cramer's Rule.

$$D=\begin{vmatrix}2&2&3\\1&-1&1\\-4&3&-2\end{vmatrix}=2(2-3)-2(-2+4)+3(3-4)=-2-4-3=-9\neq 0$$

$$D_x=\begin{vmatrix}-1&2&3\\-3&-1&1\\4&3&-2\end{vmatrix}=-1(2-3)-2(6-4)+3(-9+4)=1-4-15=-18\neq 0$$

$$D_y=\begin{vmatrix}2&-1&3\\3&-3&1\\-4&4&-2\end{vmatrix}=2(6-4)+1(-2+4)+3(4-12)=4+2-24=-18\neq 0$$

$$D_z=\begin{vmatrix}2&2&-1\\1&-1&-3\\-4&3&4\end{vmatrix}=2(-4+9)-2(4-12)-1(3-4)=10+16+1=27$$

$$\therefore x=\frac{D_x}{D}=\frac{-18}{-9}=2 \;\therefore y=\frac{D_y}{D}=\frac{-18}{-9}=2 \;\therefore x=\frac{D_z}{D}=\frac{27}{-9}=-3 \quad \therefore \bar{p}=2\bar{a}+2\bar{b}-3\bar{c}$$

(iii) $\left(\bar{a}+2\bar{b}-\bar{c}\right).\left(\bar{a}\times\bar{a}-\bar{a}\times\bar{b}-\bar{a}\times\bar{c}-\bar{b}\times\bar{a}+\bar{b}\times\bar{b}+\bar{b}\times\bar{c}\right)$

$$=\left(\bar{a}+2\bar{b}-\bar{c}\right).\left(0-\bar{a}\times\bar{b}-\bar{a}\times\bar{c}+\bar{a}\times\bar{b}+0+\bar{b}\times\bar{c}\right)$$

$$=\left(\bar{a}+2\bar{b}-\bar{c}\right).\left(\bar{c}\times\bar{a}+\bar{b}\times\bar{c}\right)$$

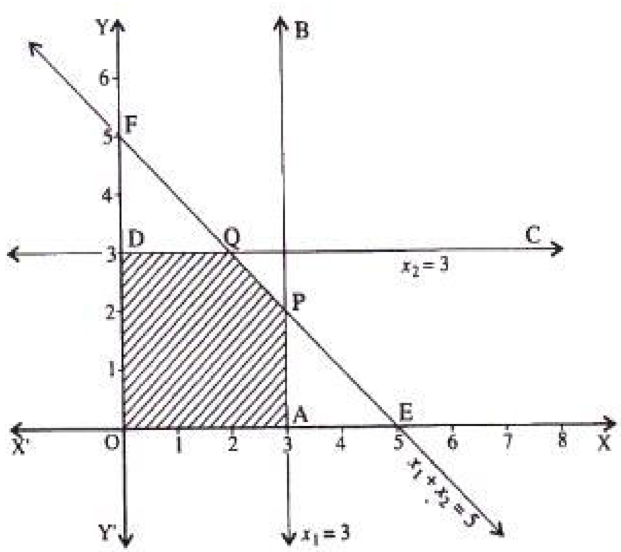
$$=\bar{a}.\left(\bar{c}\times\bar{a}\right)+\bar{a}.\left(\bar{b}\times\bar{c}\right)+2\bar{b}.\left(\bar{c}\times\bar{a}\right)+2\bar{b}.\left(\bar{b}\times\bar{c}\right)-\bar{c}.\left(\bar{c}\times\bar{a}\right)-\bar{c}.\left(\bar{b}\times\bar{c}\right)$$

$$=0+\bar{a}.\left(\bar{b}\times\bar{c}\right)+2\bar{b}.\left(\bar{c}\times\bar{a}\right)+0-0-0=\left[\bar{a}\ \ \bar{b}\ \ \bar{c}\right]+2\left[\bar{b}\ \ \bar{c}\ \ \bar{a}\right]=\left[\bar{a}\ \ \bar{b}\ \ \bar{c}\right]+2\left[\bar{a}\ \ \bar{b}\ \ \bar{c}\right]=3\left[\bar{a}\ \ \bar{b}\ \ \bar{c}\right]$$

Q.3(B) (i) First we draw the lines AB, CD and EF whose equations are x = 3, y = 3 and x + y = 5 respectively

Line	Equation	Points on the X-axis	Points on the Y-axis
AB	x = 3	A(3, 0)	-
CD	y = 3	-	D(0, 3)
EF	x + y =5	E(5, 0)	F(0, 5)

The feasible region is OAPQDO which is shaded in the figure.
The vertices of the feasible region are O(0, 0), A(3, 0),P,Q and D(0, 3).
P is the point of intersection of the lines x + y = 5 and x = 3
Substituting x = 3 in x + y = 5, we get,3 + y = 5 $\therefore y = 2$, $\therefore P \equiv (3,\ 2)$
Q is the point of intersection of the lines x + y = 5 and y = 3
Substituting y = 3 in x + y = 5, we get, x + 3 = 5 $\therefore x = 2$ $\therefore Q \equiv (2,\ 3)$



The values of the objective function z = 10x + 25y at
These vertices are
z(O) = 10(0) + 25(0) = 0
z(A) = 10(3) + 25(0) = 30
z(P) = 10(3) + 25(2) = 30 + 50 = 80
z(Q) = 10(2) + 25(3) = 20 + 75 = 95
z(D) = 10(0) + 25(3) = 75
z has maximum value 95, when x = 2 and y = 3.

ii) The normals to the planes $\bar{r}.(\bar{i}+3\bar{j}-2\bar{k})=0$ and $\bar{r}.(2\bar{i}+4\bar{j}-3\bar{k})=0$ are $\overline{n_1}=\bar{i}+3\bar{j}-2\bar{k}$ and $\overline{n_2}=2\bar{i}+4\bar{j}-3\bar{k}$ whose direction ratios are 1, 3, -2 and 2, 4, -3 respectively. Let a, b, c be the direction ratios of the line of intersection of these planes. Since the line lies in both the planes, it is perpendicular to both the normals $\overline{n_1}$ *and* $\overline{n_2}$.

$\therefore$ a + 3b - 2c = 0 and 2a + 4b - 3c = 0

$$\therefore \frac{a}{\begin{vmatrix}3 & -2\\ 4 & -3\end{vmatrix}}=\frac{-b}{\begin{vmatrix}1 & -2\\ 2 & -3\end{vmatrix}}=\frac{c}{\begin{vmatrix}1 & 3\\ 2 & 4\end{vmatrix}} \qquad \therefore \frac{a}{-9+8}=\frac{-b}{-3+4}=\frac{c}{4-6} \qquad \therefore \frac{a}{-1}=\frac{-b}{+1}=\frac{c}{-2}$$

$\therefore$ The direction ratios of the line of intersection are -1, -1, -2, i.e., 1, 1, 2.

$\therefore$ The direction cosines of the line of intersection are

$$l=\frac{1}{\sqrt{1^2+1^2+2^2}}=\frac{1}{\sqrt{6}}, \quad m=\frac{1}{\sqrt{1^2+1^2+2^2}}=\frac{1}{\sqrt{6}} \text{ and } n=\frac{2}{\sqrt{1^2+1^2+2^2}}=\frac{2}{\sqrt{6}}$$

If α,β,γ are the angles made by the line of intersection with $\bar{i},\bar{j},\bar{k}$

$$then \quad l=\cos\alpha=\frac{1}{\sqrt{6}}, \; m=\cos\beta=\frac{1}{\sqrt{6}} \text{ and } n=\cos\gamma=\sqrt{\frac{2}{3}}$$

$$\therefore \cos\alpha=\cos\beta \quad and \quad \cos\gamma=\sqrt{\frac{2}{3}} \qquad \therefore \alpha=\beta \quad and \quad \gamma=\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

Hence the line of intersection is equally inclined with $\bar{i}$ *and* $\bar{j}$ and it makes an angle of

$\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$cos with $\bar{k}$

(iii) We have to show that, $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

i.e., to show that, $\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)+\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)=\frac{9\pi}{8}$

$Let \sin^{-1}\left(\frac{1}{3}\right)=x$, $\therefore \sin x=\frac{1}{3}, where \quad 0<x<\frac{\pi}{3}$, $\therefore$ cos x > 0

$$\therefore Now, \quad \cos x=\sqrt{1-\sin^2 x}=\sqrt{1-\frac{1}{9}} \quad \therefore=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}, \quad \therefore x=\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

$$\therefore \qquad \therefore \sin^{-1}\left(\frac{1}{3}\right)=\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)\text{.............(1)}$$

$$L.H.S.=\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)+\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)=\frac{9}{4}\left[\sin^{-1}\left(\frac{1}{3}\right)+\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)\right]$$

$$=\frac{9}{4}\left[\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)+\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)\right]$$

$$=\frac{9}{4}\left[\frac{\pi}{2}\right]\text{......................}\left[\Theta \sin^{-1}x+\cos^{-1}x=\left[\frac{\pi}{2}\right]\right] \qquad =\frac{9}{8}=R.H.S.$$

SECTION II

Q.4(A)

(i) Given E(x) = 5 & Var(x) = 2.5, E(x) = np & Var(x) = npq

np = 5 & npq = 2.5, $\therefore \frac{npq}{np} = \frac{2.5}{5} \therefore q = \frac{1}{2}$, $\therefore p = 1 - q = 1 - \frac{1}{2} \therefore p = \frac{1}{2}$

$\therefore np = 5 \quad gives \quad n\left(\frac{1}{2}\right) = 5 \quad \therefore n = 10 \ \& \ p = \frac{1}{2}$ Ans (c)

(ii) Since f(x) is a p.d.f., $\int_{-\infty}^{\infty} f(x) = 1$, $\therefore \int_{-\infty}^{1} f(x) \ dx + \int_{0}^{1} f(x) \ dx + \int_{1}^{\infty} f(x) \ dx = 1$

$\therefore 0 + \int_0^1 kx^2(1-x) \ dx + 0 = 1$, $\therefore k\int_0^1 \left(x^2 - x^3\right) \ dx = 1$

$\therefore k\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 1 \ \therefore k\left[\frac{1}{3} - \frac{1}{4}\right] = 1, \quad \therefore k\left[\frac{1}{12}\right] = 1, \quad \therefore k = 12$ Ans (a)

(iii) $y = ae^{x} + be^{-3x}$

$e^{3x}y = ae^{4x} + b$ Differentiating w.r.t. x, $3e^{3x}y + e^{3x}\frac{dy}{dx} = 4ae^{4x}$

$3e^{-x}y + e^{-x}\frac{dy}{dx} = 4a$, $e^{-x}\left(3y + \frac{dy}{dx}\right) = 4a$

$-e^{-x}\left(3y + \frac{dy}{dx}\right) + e^{-x}\left(3.\frac{dy}{dx} + \frac{d^2y}{dx^2}\right) = 0$

$-3y - \frac{dy}{dx} + 3\frac{dy}{dx} + \frac{d^2y}{dx^2} = 0$

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 0$ Ans (c)

Q.4(B)

(i) Given that $f(0) = (\log 3)^2 \to (1)$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{3^x + 3^{-x} - 2}{x^2}$$

$$= \lim_{x\to 0}\frac{3^x + \frac{1}{3^x} - 2}{x^2} = \lim_{x\to 0}\frac{\left(3^x\right)^2 + 1 - 2 \ x \ \left(3^x\right) \ x \ 1}{\left(3^x\right) \ x \ x^2} = \lim_{x\to 0}\frac{\left(3^x - 1\right)^2}{\left(3^x\right) \ x \ x^2}$$

$$= \left[\left(\lim_{x\to 0}\frac{\left(3^x - 1\right)}{x}\right)^2\right].\left[\left(\frac{1}{\lim_{x\to 0} 3^x}\right)^2\right] = (\log 3)^2.\frac{1}{3^0} = (\log 3)^2 \to (2)$$

From (1) and (2), $\lim_{x\to 0} f(x) \neq f(0)$, Therefore f is continuous at x = 0

(ii) $u = e^{\log\cos 4x}$ & $v = e^{\log\sin 4x}$

$\therefore u = \cos 4x$ & $v = \sin 4x$ $\left[\because a^{\log_a x} = x\right]$

$\therefore u^2 + v^2 = \cos^2 4x + \sin^2 4x = 1$

Differentiating $u^2 + v^2 = 1$ *w.r.t. u, we get* $2u + 2v\dfrac{dv}{du} = 0$

$\therefore 2v\dfrac{dv}{du} = -2u \qquad \therefore \dfrac{dv}{du} = -\dfrac{u}{v}$

,

(iii) $1 + \dfrac{dy}{dx} = \operatorname{cosec}(x+y) \to (1)$ *put* $x + y = u$

$\therefore 1 + \dfrac{dy}{du} = \dfrac{du}{dx}$, becomes $\dfrac{du}{dx} = \operatorname{cosec} u$, $\qquad \therefore \dfrac{1}{\operatorname{cosec} u}du = dx$

Integrating we get, $\int \sin u \; du = \int dx$

$\therefore -\cos u = x + c$, $\qquad \therefore -\cos(x+y) = x + c$

$\therefore x + \cos(x+y) + c = 0$ is the general solution

(iv) $y = \cot^{-1}\left(\dfrac{1-3x^2}{3x-x^3}\right) = \tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$, *put* $x = \tan\theta, \therefore \theta = \tan^{-1} x$

$\therefore y = \tan^{-1}\left(\dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right)$, $\qquad \therefore y = \tan^{-1}(\tan 3\theta) = 3\theta$, $\qquad \therefore y = 3\tan^{-1} x$, $\qquad \therefore \dfrac{dy}{dx} = \dfrac{3}{1+x^2}$

(v) *Let* $I = \displaystyle\int_1^3 \dfrac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx$.....................(1)

We use the property $\displaystyle\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, Hence in (1), we change x by 1 + 3 - x

$\therefore I = \displaystyle\int_1^3 \dfrac{\sqrt[3]{1+3-x+5}}{\sqrt[3]{1+3-x+5}+\sqrt[3]{9-1-3+x}}\,dx = \int_1^3 \dfrac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\,dx$.....................(2)

Adding (1) & (2) we get

$2I = \displaystyle\int_1^3 \dfrac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx + \int_1^3 \dfrac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\,dx = \int_1^3 \dfrac{\sqrt[3]{x+5}+\sqrt[3]{9-x}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx = \int_1^3 1\;dx = [x]_1^3 = 3 - 1 = 2$

$\therefore I = 1$

Q. 5 (A) (i) *Let* $I = \int\sqrt{x^2+a^2}\,dx = \int\sqrt{x^2+a^2}.1dx$

$= \sqrt{x^2+a^2}.\int 1dx - \int\left[\dfrac{d}{dx}\left(\sqrt{x^2+a^2}\right)\int 1dx\right]dx = \sqrt{x^2+a^2}.x - \int\left[\dfrac{1}{2\sqrt{x^2+a^2}}(2x+0).xdx\right]$

$= \sqrt{x^2+a^2}.x - \int\left[\dfrac{x}{\sqrt{x^2+a^2}}.xdx\right] = x.\sqrt{x^2+a^2} - \int\left[\dfrac{x^2+a^2-a^2}{\sqrt{x^2+a^2}}dx\right]$

$$= x.\sqrt{x^2+a^2} - \int \sqrt{x^2+a^2}dx + a^2\int\left[\frac{dx}{\sqrt{x^2+a^2}}\right] = x.\sqrt{x^2+a^2} - I + a^2\log\left|x+\sqrt{x^2+a^2}\right| + c_1$$

$$\therefore 2I = x\sqrt{x^2+a^2} + a^2\log\left|x+\sqrt{x^2+a^2}\right| + c_1 \quad \therefore I = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + \frac{c_1}{2}$$

$$\therefore \int\sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c \quad where \quad c = \frac{c_1}{2}$$

(ii) $Let\ I = \int x^5.\sqrt{a^3+x^3}dx = \int x^2.x^3.\sqrt{a^3+x^3}dx$, $Put \quad x^3 = t \therefore 3x^2dx = dt$

$$= \int t.\sqrt{a^3+t}.\frac{dt}{2} \quad = \frac{1}{2}\int\left(t+a^3-a^3\right)\sqrt{a^3+t}.dt \quad = \frac{1}{2}\left[\int\left(t+a^3\right)\sqrt{a^3+t}.dt - \int a^3\sqrt{a^3+t}.dt\right]$$

$$= \frac{1}{2}\left[\int\left(a^3+t\right)^{3/2}dt - a^3\int\left(a^3+t\right)^{1/2}.dt\right] = \frac{1}{2}\left[\frac{\left(a^3+t\right)^{5/2}}{\frac{5}{2}} - a^3\frac{\left(a^3+t\right)^{3/2}}{\frac{3}{2}}\right] + c \quad \text{Resubstituting t,}$$

$$I = \frac{\left(a^3+x^3\right)^{5/2}}{5} - a^3\frac{\left(a^3+x^3\right)^{3/2}}{3} + c$$

(iii) Let X = no of trials, p = no. of successes, q = no. of failures

Given : np = 3 & npq = (3/2), $\therefore q = \frac{1}{2} \therefore p = 1-q = 1-\frac{1}{2} \therefore p = \frac{1}{2}$

$$\therefore n.\frac{1}{2} = 3 \therefore n = 6, \quad \therefore x \sim B\left(6, \frac{1}{2}\right),$$

The p.m.f. of X is given by P[X = x] = ${}^nC_x \quad p^x \quad . \quad q^{n-x}$ i.e. P[x] = ${}^6C_x \left(\frac{1}{2}\right)^x . \left(\frac{1}{2}\right)^{6-x}$

P(at least 4 successes) $= P[X \geq 4] = P[X = 4] + P[X = 5] + P[X = 6]$

$$P(x=4) = {}^6C_4 \left(\frac{1}{2}\right)^4 . \left(\frac{1}{2}\right)^2 = 15\left(\frac{1}{2}\right)^6 = \left(\frac{15}{64}\right)$$

$$P(x=5) = {}^6C_5 \left(\frac{1}{2}\right)^5 . \left(\frac{1}{2}\right)^1 = 6\left(\frac{1}{2}\right)^6 = \left(\frac{6}{64}\right)$$

$$P(x=6) = {}^6C_6 \left(\frac{1}{2}\right)^6 . \left(\frac{1}{2}\right)^0 = 1\left(\frac{1}{2}\right)^6 = \left(\frac{1}{64}\right)$$

Therefore P(at least 4 successes) $= \frac{15}{64} + \frac{6}{64} + \frac{1}{64} = \frac{22}{64} = \frac{11}{32}$

Q.5 (B) (i) f is continuous on [0, 8] $\therefore$ f is continuous at x = 2 and x = 4.

Continuity at x = 2 for

$$f(x) = x^2 + ax + 6, \quad for \quad 0 \leq x < 2$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}\left(x^2+ax+6\right) = (2)^2 + 2(a) + 6 = 2a + 10 \quad f(x) = 3x+2, \quad for \quad 2 \leq x \leq 4 ....(1)$$

$f(2) = 3(2) + 2 = 8$, 'f' is continuous at x = 2, $\therefore \lim_{x\to 2^+} f(x) = f(2) \quad \therefore 2a + 10 = 8 \quad \therefore 2a = -2 \therefore a = -1$, Continuity at x = 4

From (1), $f(4)=3(4)+2=14$

$f(x)=2ax+5b, \quad for \quad 4<x\le 8$

$\therefore \lim_{x\to 4^+} f(x)=\lim_{x\to 4^+} f(2ax+5b) = 2a(4)+5b=8a+5b$

'f' is continuous at x = 4, $\therefore \lim_{x\to 4^+} f(x)=f(4)$, 8a + 5b = 14,

-8 + 5b = 14……… $[a=-1]$, b = 22/5

(ii) Solution $\frac{dy}{dx}+2y\tan x=\sin x$..........(1)

This is the linear differential equation of the form $\frac{dy}{dx}+py=Q, \quad where \quad P=2\tan x \quad and \quad Q=\sin x$

$I.F.=e^{\int Pdx}=e^{\int 2\tan x dx}.=e^{2\log\sec x}=e^{\log\sec^2 x}=\sec^2 x$

Multiplying both sides of (1) by I.F. $\sec^2 x$ we get,

$$\sec^2 x\left(\frac{dy}{dx}+2y\tan x\right)=\sec^2 x.\sin x$$

$$\therefore \frac{d}{dx}\left[(\sec^2 x).y\right]=\sec^2 x.\sin x$$

Integrating both the sides w.r.t. x, we get, $(\sec^2 x).y=\int \sec^2 x.\sin x dx+C$

$\therefore y.\sec^2 x=\int\frac{\sin x}{\cos^2 x}dx+C$ put cos x = t, $\therefore -\sin x dx=dt \qquad \therefore \sin x dx=-dt$

$\therefore y.\sec^2 x=\int\frac{1}{t^2}(-dt)+C \qquad \therefore y.\sec^2 x=-\int t^{-2}(dt)+C \qquad \therefore y.\sec^2 x=-\frac{t^{-1}}{-1}+C$

$\therefore y.\sec^2 x=\frac{1}{t}+C \qquad \therefore y.\sec^2 x=\frac{1}{\cos x}+C \qquad \therefore y.\sec^2 x=\sec x+C$ This is the general solution.

$Now, y=0 \quad when \quad x=\frac{\pi}{3} \qquad 0=\sec\frac{\pi}{3}+c \qquad 0=2+c \therefore C=-2$,

The particular solution is $\therefore y.\sec^2 x=\sec x-2$

(iii) $f(x)=x^2+\frac{16}{x^2}$, $\therefore f'(x)=\frac{d}{dx}(x^2)+16\frac{d}{dx}(x^{-2}) = 2x+16(-2)x^{-3}=2x-\frac{32}{x^3}$

$and \quad f''(x)=\frac{d}{dx}(2x)-32\frac{d}{dx}(x^{-3}) = 2x1-32(-3)x^{-4}=2+\frac{96}{x^4}$

$\therefore f'(x)=0 \quad gives \quad 2x-\frac{32}{x^3}=0 \qquad \therefore 2x^4-32=0 \therefore x^4=16 \therefore x\pm 2$

$\therefore the \quad roots \quad of \quad and \quad f'(x)=0 \quad are \quad x_1=2 \quad and \quad x_2=-2$

$$f''(2)=2+\frac{96}{(2)^4}=8>0$$

Therefore by second derivative test,'f' has minimum at x = 2 and minimum value of 'f' at x = 2.

$$=f(2)=(2)^2+\frac{16}{(2)^2}=4+4=8$$

$$f''(-2)=2+\frac{96}{(-2)^4}=8>0$$

Therefore by second derivative test,'f'has minimum at x = - 2 and minimum value of'f'at x=- 2.

$$=f(-2)=(-2)^2+\frac{16}{(-2)^2}=4+4=8$$

Hence, the function 'f' has minimum value 8 at $x=\pm 2$

Q. 6 **(A)** (i) $I=\int \frac{2x^3+3x^2-3}{2x^2-x-1}\ dx=$

$$(2x^3+3x^2-3)=(2x^2-x-1).(x+2)+(3x-1)........(By\quad Factorisation)$$

$$\therefore I=\int \frac{(2x^2-x-1).(x+2)+(3x-1)}{2x^2-x-1} \qquad \therefore I=\int\left[(x+2)+\frac{(3x-1)}{2x^2-x-1}\right]\ dx$$

$$\therefore I=\int\left[(x+2)+\frac{(3x-1)}{(2x+1)(x-1)}\right]\ dx \qquad =\int(x+2)dx+\int\left[\frac{(3x-1)}{(2x+1)(x-1)}\right]\ dx$$

$$Let\frac{(3x-1)}{(2x+1)(x-1)}=\frac{A}{(2x+1)}+\frac{B}{(x-1)}$$

$$\therefore (3x-1)=A(x-1)+B(2x+1)$$

$put\quad (2x+1)=0\quad i.e.\quad x=\frac{-1}{2}\quad we\quad get\quad 3\left(\frac{-1}{2}\right)-1=A\left(\frac{-1}{2}-1\right)+B\left[2\left(\frac{-1}{2}\right)+1\right]$

$$\therefore\left(\frac{-3-2}{2}\right)=A\left(\frac{-1-2}{2}\right)+B(0) \qquad \therefore -5=A(-3)\therefore A=\left(\frac{5}{3}\right)$$

$put\quad (x-1)=0\quad i.e.\quad x=1\quad we\quad get\quad 3(1)-1=A(0)+B[2(1)+1] \qquad \therefore 2=B(3) \qquad \therefore B=\left(\frac{2}{3}\right)$

$$\therefore\int\left[\frac{(3x-1)}{(2x+1)(x-1)}\right]\ dx\ =\int\left[\frac{\frac{5}{3}}{(2x+1)}\right]\ dx+\int\left[\frac{\frac{2}{3}}{(2x+1)}\right]\ dx$$

$$\therefore I=\int(x+2)dx+\frac{5}{3}\int\frac{dx}{(2x+1)}+\frac{2}{3}\int\frac{dx}{(x-1)} \qquad \therefore I=\frac{(x+2)^2}{2}+\frac{5}{3}\frac{\log|2x+1|}{2}+\frac{2}{3}\log|x-1|+C$$

(ii) $\int_0^{\pi/4}\frac{dx}{3\cos 2x+5}\ dx\quad put\quad \tan x=t,\therefore x=\tan^{-1}t$,

$$\therefore dx = \frac{dt}{1+t^2}, \quad \cos 2x = \frac{1-t^2}{1+t^2}, \quad \text{When x = 0, t = 0;} \quad x = \frac{\pi}{4}, \quad t = 1$$

$$\therefore I = \int_0^1 \frac{\frac{dt}{1+t^2}}{\frac{3(1-t^2)}{1+t^2}+5} = \int_0^1 \frac{\frac{dt}{1+t^2}}{\frac{3-3t^2+5+5t^2}{1+t^2}}$$

$$= \int_0^1 \frac{dt}{2t^2+8} = \frac{1}{2}\int_0^1 \frac{dt}{t^2+4} \quad \therefore \frac{1}{2}\left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right]_0^1 = \frac{1}{4}\left[\tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}(0)\right]$$

$$= \frac{1}{4}\left[\tan^{-1}\left(\frac{1}{2}\right)\right]$$

(iii) $Let \quad I = \int_{\pi/2}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x}\right) \ dx \qquad = \int_{\pi/2}^{\pi} e^x \left(\frac{1}{1-\cos x} - \frac{\sin x}{1-\cos x}\right) \ dx$

$$= \int_{\pi/2}^{\pi} e^x \left[\frac{1}{2\sin^2\left(x/2\right)} - \frac{2\sin\left(x/2\right)\cos\left(x/2\right)}{2\sin^2\left(x/2\right)}\right] dx \qquad = \int_{\pi/2}^{\pi} e^x \left[\cot\left(x/2\right) - \frac{1}{2}\operatorname{cosec}^2\left(x/2\right)\right] dx$$

$put \quad f(x) = \cot\left(x/2\right)$

$$\therefore f'(x) = -\operatorname{cosec}^2\left(x/2\right) \cdot \frac{1}{2} = -\frac{1}{2}\operatorname{cosec}^2\left(x/2\right)$$

$$\therefore I = -\int_{\pi/2}^{\pi} e^x \left[f(x) + f'(x)\right] \ dx$$

$$= -\left[e^x . f(x)\right]_{\pi/2}^{\pi} = -\left[e^x . \cot\left(x/2\right)\right]_{\pi/2}^{\pi}$$

$$= -\left[e^{\pi}.\cot\left(\pi/2\right) - e^{\pi/2}.\cot\left(\pi/4\right)\right] = -\left[e^{\pi} x \quad 0 - e^{\pi/2} x \quad 1\right] \qquad = \left[e^{\pi/2}\right]$$

Q.6 (B) (i) $P(x<1.5) = \int_{-\infty}^{1.5} f(x) \ dx \quad = \int_{-\infty}^{0} f(x) \ dx + \int_{\infty}^{1.5} f(x) \ dx$

$$= 0 + \int_0^{1.5} f(x) \ dx \left[\because f(x) = 0 \, for \quad x<0\right] = \int_0^{1.5} \frac{1}{2} dx = \frac{1}{2}\int_0^{1.5} dx = \frac{1}{2}[x]_0^{1.5} \quad = \frac{1}{2}[1.5-0] = \frac{1.5}{2} = 0.75$$

$$P(x>1) = \int_1^{\infty} f(x) \ dx = \int_1^2 f(x) \ dx + \int_2^{\infty} f(x) \ dx = \int_1^2 f(x) \ dx + 0 \left[\because f(x) = 0 \, for \quad x>2\right]$$

$$= \int_1^2 \frac{1}{2} dx = \frac{1}{2}\int_1^2 dx = \frac{1}{2}[x]_1^2 \ = \frac{1}{2}[2-1] = \frac{1}{2} = 0.5$$

(ii) Let $\partial u \quad and \quad \partial y$ be the increments in u and y respectively,

corresponding to the increment dx in x. Now y is a differentiable function of u and v is a differentiable function of x

$$\therefore \frac{dy}{du} = \lim_{du \to 0} \frac{\partial y}{\partial u} \quad and \quad \frac{du}{dx} = \lim_{dx \to 0} \frac{\partial u}{\partial x} \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$Also \lim_{dx\to 0} du = \lim_{dx\to 0}\left(\frac{du}{dx}.dx\right) = \left(\lim_{\partial X\to 0}\frac{\partial u}{\partial x}\right)\left(\lim_{\partial x\to 0}\partial x\right) = \frac{du}{dx}. \quad 0 = 0$$

This means that as $\partial x \to 0, \partial u \to 0$......................(2)

Now, $\frac{\partial y}{\partial x} = \frac{\partial y}{\partial u}.\frac{\partial u}{\partial x}$......................$(\partial u \neq 0)$

Taking limits as $\partial x \to 0$, *we get*

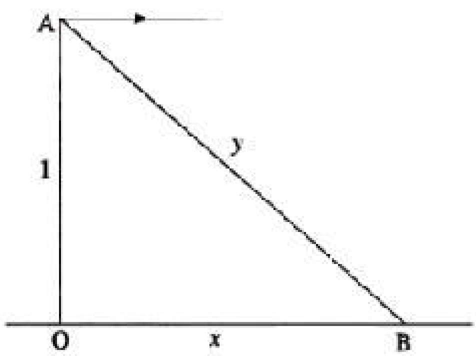
$$\left(\lim_{\partial x\to 0}\frac{\partial y}{\partial x}\right) = \left(\lim_{\partial x\to 0}\frac{\partial y}{\partial u}\cdot\frac{\partial u}{\partial x}\right) = \left(\lim_{\partial u\to 0}\frac{\partial y}{\partial u}\cdot\lim_{\partial x\to 0}\frac{\partial u}{\partial x}\right) = \left(\lim_{dU\to 0}\frac{\partial y}{\partial u}\cdot\lim_{dx\to 0}\frac{\partial x}{\partial x}\right)\ldots\ldots\ldots[By(2)]$$

Now both the limits on R.H.S. exists ………………..[By (1)] $\therefore \lim_{dx\to 0}\frac{\partial x}{\partial x}$ *exists and is equal to* $\frac{dy}{dx}$

$$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots[By(1)]$$

$$y = \sin(x^2+5) \therefore \frac{dy}{dx} = \frac{d}{dx}\left[\sin(x^2+5)\right] = \cos(x^2+5).\frac{d}{dx}(x^2+5) = \cos(x^2+5).(2x)$$

(iii)



Let A be the position of aeroplane and B be the position of observerat time t. Given the altitude of the plane, i.e., OA = 1 km. Let OB = x,
the horizontal distance and AB = y, the actual distance of the aeroplane from the observer at time t.
Then $\frac{dx}{dt} =$ 800km/hr is the rate at which the aeroplane is moving over an observer and $\frac{dy}{dt} =$ is the rate at which the aeroplane is approaching to the observer.
From the figure, $y^2 = x^2 + 1$........(1)

Differentiating w.r.t. t, we get, $2y\frac{dy}{dt} = 2.\frac{dx}{dt} + 0$ $\quad \therefore \frac{dy}{dt} = \frac{x}{y}.\frac{dx}{dt}$..........(2)

when y = 1250 metres = $\frac{1250}{1000}km$ then from (1), we get, $\frac{25}{16} = x^2 + 1 \therefore x^2 = \frac{9}{16} \therefore x = \frac{3}{4}$,

$$\therefore (2) gives, \frac{dy}{dt} = \frac{3/4}{5/4}.800 = \frac{3}{5}.800 = 480 \quad km/hr$$

Therefore the aeroplane is approaching to the observer at the rate of 480 km/hr.